Lab 2

Newton's Second Law and forces

A. Purpose

To determine the velocity and the acceleration of a moving cart on an inclined track by Multi-functional Counter and Photogates.

B. Introduction

If an object is subject to external forces, the velocity will change. Experiments show that for a given object, the acceleration is linearly proportional to the net force, which in mathematical description is

$$F \propto a \tag{1}$$

Moreover, experiments also show that under a given force, the acceleration is inversely linearly proportional to the intrinsic quantity of the object, mass, which in mathematical description is

$$\mathbf{a} \propto \frac{\mathbf{F}}{m} \tag{2}$$

Therefore, we conclude that

$$\mathbf{F} = k \cdot m \cdot \mathbf{a} \tag{3}$$

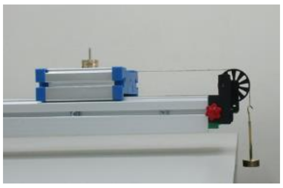
where m represents the mass of the object, and k is a proportionality constant. It is our choice to take k to be 1. All it changes is merely how we define mass. Thus, Newton's 2nd law is simply that

$$\sum_i \mathbf{F_i} = m\mathbf{a} \tag{4}$$

Note that the inertia of a body is its tendency to resist any change in its state of motion; therefore, the mass of a body is a measure of its inertia, that is, its resistance to change in velocity.

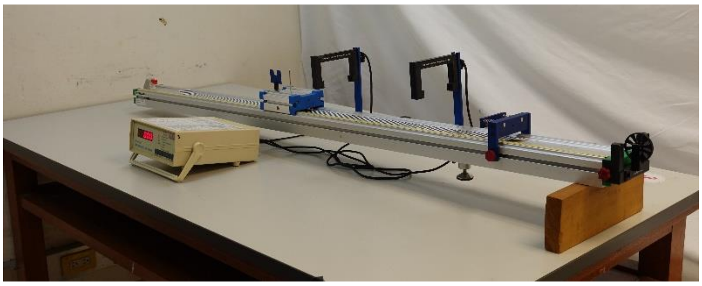
This experiment applies the understanding of Newton's laws to determine the unknowns in the laboratory apparatus.

C. Apparatus and software

			
Cart and track with pulley and hanger	Multi-functional Counter	Photogates	Level

D. Procedures

1. Pre-lab assignments (hand in before the lab)
   (1) Read the instructions for use of the multi-functional counter carefully to learn how to use it to measure the quantities.
   (2) Make a flowchart of this experiment and answer the questions:
   (i) During the experiment, you will have the apparatus consisting of a cart of unknown mass M, situated on a track, connected by a string to a hanging mass m, as shown in Fig. 1. A block is placed under the track to tilt it by a small unknown angle θ, where friction is small but significant, illustrated in Fig. 2. When the cart is released, the cart accelerates, the pulley turns, and the multi-functional counter with two photogates will measure the velocities of the cart at two different positions, by which it obtains the acceleration of the cart.



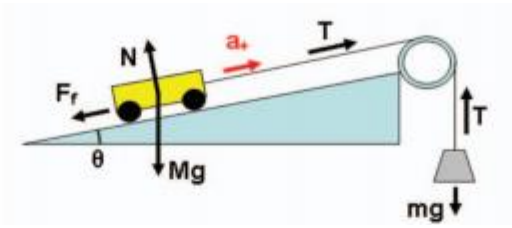


Fig. 1. The experimental setup. Fig. 2. Schematic of the experiment

(a) When $mg > Mg\sin\theta$, the cart moves up the sloping track with an acceleration $a_+ > 0$, and instead descends the slope with the acceleration $a_- < 0$ for smaller values of m. If a_+ is along the positive direction and the effective friction force is regarded as a constant force $|\mathbf{F_f}|$ opposing the motion, prove that

$$a_\pm(m) = \frac{m - M\sin\theta \mp |\mathbf{F_f}|/g}{m+M}\,g \tag{5}$$

(b) Following (a), you can imagine that the acceleration of the cart with changing hanging mass will be nicely described by two straight lines if $M \gg m$, with x-intercepts at m_+ and m_- where $a_\pm = 0$. Draw a graph of the acceleration a versus the hanging mass m. Specify the positions of $m_\pm$, and find $|\mathbf{F_f}|$ in terms of $m_\pm$.

(c) Following (b), find the mass of the cart M by approximating the slope of a_- with $M \gg m$, and find the angle θ by the results you obtained in terms of $m_\pm$.

(d) Suppose the track is now horizontal without a block under it, find a simple way to obtain the friction $|\mathbf{F_f}|$ using the multifunctional counter and the two photogates.

2. In-lab activities

(1) Acceleration of a cart on an inclined track

(i) Place a block under the track to tilt it by a small angle θ as Fig. 1 shows.

(ii) Mount the end stop to the track just in front of the pulley.

(iii) Determine the length of string to have the mass hanger be near the floor when the cart reaches the end stop of the track. One end of the string should be attached to the rod on the cart; the other end of the string should be tied to the mass hanger.

(iv) Adjust the angle of the pulley so that the string is parallel to the track. **(Why?)**

(v) To find $|a_\pm|$ by Multi-functional counter with Photogates

(a) Fix two photogates at the proper positions. (roughly 25 cm apart in distance)

(b) Press **FUNCTION** to change the mode to "Acceleration," where you can get the velocities at two positions, and the average acceleration.

(c) Press **CHANGEOVER** for more than 1 second to set $3.0\,\text{cm}$ for U-shaped baffle plate.

(d) Hold the cart still at the position right before the first photogate, and then release it to obtain data.

(e) Press **FUNCTION** to remove data. Redo the experiment 3 times to obtain the best estimate for the acceleration $a_\pm$.

(vi) Do the trials of hanging mass being

$$1, 2, 3, 4, \ldots\ldots, 28, 29, 30, 40, 60, 80, 100\text{ g}.$$

Each metal gasket has mass of about 1 g and each weight has that of about 10 g. **(Measure by yourself using the electronic balance.)** Use eq(5) to determine the mass of the cart M, the tilted angle θ, and the effective friction force $|\mathbf{F_f}|$.

(2) Measure the mass of the cart M by the electronic balance, and the tilted angle θ using the geometry. (Hint: trigonometric ratio) Use the simple way you proposed in the pre-lab assignment to obtain the effective friction force $|\mathbf{F_f}|$. Compare the results with part 1.

3. Post-lab report

(1) Recopy and organize your data from the in-lab tables in a neat and more readable form.
(2) Analyze the data you obtained in the lab and answer the given questions

E. Questions

1. Try to obtain the coefficient of friction between the cart and the track and state it in the standard form. Is the result the static or the kinetic friction coefficient between them? Why?
2. In the pre-lab assignments, you find the mass of the cart M by assuming $M \gg m$. Can you avoid using the approximation that $M \gg m$? (Hint: fitting by "cftool" in Matlab.)
3. With the approximation $M \gg m$, one finds the acceleration of the cart with changing hanging mass is nicely described by two straight lines, where $|\mathbf{F_f}|$ can be determined by finding the x-intercepts ($m_{\pm}$) of two lines. If we now instead define the y-intercepts (a_0^+ and a_0^-) of two straight lines,
   (1) prove that the inclined angle θ as well as the effective friction force $|\mathbf{F_f}|$ can be described by a_0^+ and a_0^-.
   (2) Is it appropriate to use the y-intercepts (a_0^+ and a_0^-) of two straight lines to describe the inclined angle θ or the effective friction force $|\mathbf{F_f}|$? If so, what are the physical interpretations of a_0^+ and a_0^-? If not, explain why.
4. **(Optional)** During the experiment, it is important to make sure the track is horizontal. If now the table is, however, not level but tilted by an angle α, the results ($|\mathbf{F_f}|$, M, θ) you obtained by the x-intercepts ($m_{\pm}$) of two lines, should be modified.
   (1) How to modify the values of the results ($|\mathbf{F_f}|$, M, θ)?
   (2) How to measure the angle α in the lab by a long ruler and a level? Explain.
   (3) If we instead conduct this experiment on the moon, what are the differences?